Behavior of Particle Tracers in the Steady and Periodic Wakes of a Circular Cylinder

Shaswat Saincher and Jyotirmay Banerjee

Abstract—Tracer particles introduced into the flow domain for experimental flow visualization often tend to exhibit chaotic behavior even for the simplest of configurations. These passive tracers do not exactly follow the regime in general. The spatial distributions formed by such particles within the flow field and the probable trajectories followed by them are governed by the phenomenon of Lagrangian chaos. Since the problem at hand has its foundations in the theories of Hamiltonian dynamics and deterministic chaos, drawing a relation between the Eulerian flow field and the Lagrangian tracer field is a difficult task. However, analyzing this behavior would provide useful insight in interpreting the results of flow visualization experiments which involve particle tracking. Based on this background, the phenomenon of chaotic advection for a laminar cross-flow over a circular cylinder has been investigated numerically. Two configurations are considered, one involving a steady flow at Re=20 while the other characterizing a periodic shedding regime at Re=100. Computational visualization of the flow is carried out by locally injecting neutrally buoyant particles at zero momentum and tracking the same by FLUENT’s discrete phase model. The tracks made by the tracers, being equivalent to streaklines, are generated for different injection locations and the trajectories have been compared with the streamline patterns. It has been observed that the steady regime at Re=20 does not exhibit chaotic advection and the streaklines coincide with the streamlines. On the other hand, the shedding regime, being temporally dependent, depicts a high degree of tracer mixing. The particles tend to form complex spatial distributions in this case. Further, the formation of a particular tracer distribution has been observed to be strongly dependent on the location of the injector within the domain. Since the tracer particles exhibit stochastic behavior for an otherwise simple and periodic underlying flow, it is expected that such patterns can be misleading. A careful study of all possible tracer distributions is hence advisable for visualization of such time dependent flows.

Index Terms—Cylinder, discrete phase model, Lagrangian chaos, passive tracers, visualization

I. INTRODUCTION

The case of a circular cylinder placed in a cross-flow is being studied extensively for the past five decades. This is because the case is extremely significant from an engineering point of view. Further, in spite of the fact that the geometry is simple to conceive both experimentally as well as computationally, the configuration exhibits a rich variety of flow phenomena spanning over a range of Reynolds numbers. These regimes can be broadly classified as depicted in the following paragraph.

Below a Reynolds number of 4, the flow is completely attached. Above this value the flow separates from the cylinder surface forming a pair of counter rotating vortices adjacent to the rear stagnation point [1]. The regime is stable to all three dimensional disturbances. Beyond Re $\approx$ 49, instability sets in the wake by virtue of a Hopf bifurcation [2] and vortices are alternately shed from the top and bottom stagnation points on the cylinder periphery. The flow becomes unsteady but not unstable. It is merely that by alternate shedding of vortices, the flow attains a new, stable configuration. Two shedding modes have been identified in this case which are in turn dependent on the spanwise boundary conditions. One is the commonly observed parallel shedding mode while the other involves oblique shedding in which the vortices are shed at an angle to the spanwise direction. However, the laminar shedding regime persists only until Re $\approx$ 194 beyond which transition to three dimensionality takes place [2]. The transition takes place through two discontinuities or ‘modes’, the first being a mode A instability which sets in around a Reynolds of 194 [2]. This involves the formation of spanwise vortex loops which pair in the streamwise direction. A further increase in the Reynolds number in the range of 230 to 250 results in a mode B shedding. The regime is characterized by large scale spot - like vortex dislocations which lead to low frequency fluctuations in the velocity magnitude. Beyond a Reynolds number of 260, starting from the wake, the flow begins a transition to turbulence. This is characterized by an increase in the formation length (Lf) of the wake with increasing Reynolds number which is observable up to Re 1000. Beyond this point, the value of Lf reduces due to shear layer instabilities and eventual roll - up of turbulent vortices [1]. The transition to turbulence in the free shear layer begins at this stage with the inception of Kelvin - Helmholtz waves at Re $\approx$ 1300. The flow becomes turbulent, but not completely random, with the Karman shedding regime still observable in the downstream region. The boundary layers on the surface of the cylinder eventually become turbulent beyond a Reynolds number of $3 \times 10^5$.

Through experimentation, the existence of these stable and unstable regimes was suggested by Roshko in as early as 1954 [3]. His study was followed by a host of research works [4]-[10]. The annual review [2] is an excellent summation of the progress that has been made in this direction in the last four decades. However, the smoke visualizations of the Karman vortex street discussed in [5] and [6] hold special significance. Both studies grossly misinterpreted the streak line pattern obtained as a result of smoke visualization and theorized the Karman vortex street to be present at all downstream
distances. It is important to note here that the presence of elongated Karman vortices at distances as great as 600D [6] certainly violates the viscous dissipation of vorticity. This idea was challenged by Cimbala in his Ph.D. thesis [11] of 1984. He established that contemporary methods of streak line generation and interpretation were incorrect. It was argued that the introduction of smoke just downstream of the cylinder was acceptable; but that realization itself was not sufficient for the visualization of the entire downstream flow. It was because of the integration effect of streak lines that the smoke particles incorrectly indicated vortices far downstream of the cylinder. The study explained that the smoke particles have an integrated memory associated with them. Hence the incorrect vortical structure indicated by the tracers in the far wake was actually because of the particles’ past history. Cimbala found that by introducing the smoke further downstream resulted into a streakline pattern which was in accordance with the actual streamtraces. There were no complex convolutions in the tracer field thereby correctly indicating a very weak vortical behavior far downstream and thus satisfying the viscous dissipation of vorticity.

What was observed in 1984 [11] was the phenomenon of Lagrangian chaos or chaotic advection. The phenomenon can be broadly defined as the stochastic behavior of passive tracers in a fluid flow. Since then, chaotic advection has been a topic of extensive research. The knowledge hence gained has provided useful insight in addressing discrepancies found in the streamline and streakline patterns of flow visualization experiments.

Stemming from this background, the present investigation focuses on the numerical visualization of a two-dimensional cross-flow over a circular cylinder using particle tracers. Two distinct flow regimes have been numerically simulated. The first case is at a Reynolds number of 20 which falls into Roshko’s steady and laminar range [3]. The second case, falling within Roshko’s stable range [3] at a Reynolds number of 100, characterizes a periodic shedding of vortices in the downstream region. Simulations are carried out in a rectangular domain which is 120D long and 10D high (where D is the diameter of the cylinder). A long domain has been intentionally selected in order to capture a complete dissipation of vortices by viscosity. In order to simulate visualization; passive tracer particles have been injected at multiple locations downstream of the cylinder. The injection is made after the start-up transients associated with the initialization of calculations exit the domain. A particle tracking scheme has been adopted using the discrete phase model (DPM) to precisely compute the spatial trajectories of the tracers. The resultant particle tracks are compared with streamlines for both cases and an attempt has been made to explain the behavior mathematically. It is finally established that realizations from multiple particle injections require consideration if a complete understanding of the underlying flow is to be obtained.

II. NUMERICS

The computational domain along with the boundary conditions considered for simulation is shown in figure 1. The domain is 14.5D upstream and 104.5D downstream with a height of 10D. The flow is driven by a constant velocity at the inlet and a zero gauge pressure assigned as the outlet. The top and bottom edges of the domain are kept symmetric in order to establish a free - stream boundary condition. A non-uniform, structured, quadrilateral mesh has been generated in the domain. A boundary layer of progressively increasing thickness has been created around the cylinder for 20 cell layers with the first grid point being 0.0001D away from the surface of the cylinder. The solution is obtained using a laminar solver based on the SIMPLE algorithm. The time marching is carried out with a second-order implicit scheme with a non-dimensional time step ($\Delta t/DU/\infty$) of 0.01. A second order upwind scheme is used for the advection terms. Shedding is initiated by giving a perturbation of $\pm 0.2 U/\infty$ in the $u_y$ component of velocity. The unsteady calculations are made up to a unity residence time of the fluid particles in the domain. Local injections of simulated tracers have been made using FLUENT’s discrete phase model (DPM). The discrete second phase is simulated in a Lagrangian frame of reference. The particles are considered as neutrally buoyant with respect to the continuous phase ($\rho_p = \rho_{fluid} = 0.1kg/m^3$). This ensures a good tracking performance. The tracers are assumed to be spherical particles having a diameter of 5. Selection of such a size is justified [12] since the tracers are rendered large enough to avoid Brownian motion and small enough to be easily carried by the flow. The trajectory of a particle tracer is predicted by integrating the force balance [13] in the discrete phase, written in a Lagrangian reference frame (for the x-direction) as,

$$\frac{du_p}{dt} = \frac{F_D}{m_p} (u - u_p) + \frac{g (\rho_p - \rho_{fluid})}{\rho_p} + \frac{F_x}{m_p}$$

The particles are introduced at zero momentum. Further, the negligible mass and inertness of the tracers implies that the particles are bound to follow the flow exactly. Hence, one may state that velocity of the continuous phase = velocity of the discrete phase ($u = u_p$). Thus, equation (1) is modified

![Flow domain and boundary conditions with injector locations.](image)
as,
\[
\frac{du_p}{dt} = F_D (u - u_p) + g_x (\rho_p - \rho_f) \frac{d}{d} + F_x \tag{2}
\]

Equation set (3) governs the motion of the tracer particles which exist in hydrodynamic equilibrium with the continuous phase and are neutrally buoyant. The numerical solution of equation (3) is performed by employing an unconditionally stable, implicit, Euler integration scheme. The selection of this scheme is complementary to the simplicity of the case under consideration and the existent hydrodynamic equilibrium [13].

Particle injections have been made at one upstream (group) and two downstream (point) locations as indicated in figure 1. A group injection is made in order to mimic soap-film visualization with an essentially 2-D flow [2]. This is helpful in achieving a global understanding of the behavior of particle tracers in the shedding regime. Further, variations in the behavior of the tracer particles can be studied by making similar particle injections in regions which differ significantly in vortical strength. Thus, referring fig. 1, injector 1 is located in the wake region, while, injector 2 lies in a region further downstream which is devoid of significant vortical activity.

### III. RESULTS AND DISCUSSION

The results obtained for the steady regime at Re = 20 are discussed first. The analysis involves a comparison between the streamline pattern and the particle trajectories. This is followed by an overview of the results obtained for the shedding regime. Details of a quantitative validation obtained with the experimental results of [3] and the numerical findings of [14] are initially mentioned. Further, the particle tracks generated by the group injection are compared with streamtraces and with the soap-film visualization of [15]. Finally, an attempt has been made to explain the tracer behavior for both the cases; by applying the advection equations.

1) Case 1: Re = 20

Streamlines in the near-field of the cylinder are shown in fig. 2 (a) for the case of cross-flow at Re = 20. As explained in [3], the regime characterizes a pair of standing vortices, in the immediate region, downstream of the cylinder. The vortices are a counter-rotating pair and hence establish a region in which the vorticity dominates the strain-rate. Once this stable configuration is reached, the regime becomes temporally invariant and viscous effects become prevalent. The streamlines, in the region downstream of the standing vortices, are parallel to the domain axis (streamwise direction).

Fig. 2 (b) shows the release of passive tracers from the group injection. Once released, the ash particles are immediately caught up in the viscous wakes and eventually remain localized to this region for high residence times. The tracks mimic a standing vortex. Further, the particles tend to accumulate at a fixed radial distances from the center of the vortex. These distances denote locations at which the inward pressure gradient and the centrifugal force balance. The particle tracks obtained in this study, match in behavior to the 2-D soap film visualization of [15] which belongs to the steady range (Re < 49). This is shown in fig. 2(c). A fundamental interpretation which can be drawn from these findings is that the particle behavior lacks stochasticity. Hence, this case represents a classical example of a visualization experiment in which the streamline pattern exactly matches the streamlines.

2) Case 2: Re = 100

The second case deals with flow over a circular cylinder at Re=100. The value of Re suggests that the regime displays periodic shedding of vortices in the downstream region. Figure 3 shows the variation of lift and drag coefficients for this case.
Fig. 3. Temporal variation of the lift and drag coefficients at Re=100.

TABLE I

COMPARISON OF NUMERICAL AND EXPERIMENTAL DATA

<table>
<thead>
<tr>
<th></th>
<th>Present study</th>
<th>Literature</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(CD)_{avg}$</td>
<td>1.34</td>
<td>1.28 (Braza et al. 1986, Sheard et al. 2005)</td>
</tr>
<tr>
<td>$(Cl)_{rms}$</td>
<td>0.1128</td>
<td>N.A.</td>
</tr>
<tr>
<td>$St$</td>
<td>0.16</td>
<td>0.16 (Roshko 1954, Braza et al. 1986)</td>
</tr>
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Results obtained from the laminar solver are compared with the experimental and numerical data available in literature [3], [14] and [16]. The comparison is shown in the table I.

The average drag coefficient computed here deviates from the experimental results by 4.688 % due to the two-dimensionality of the present computations. The Strouhal number, however, matches exactly. Also, the frequency of oscillations in the computed drag coefficient is twice that of the lift coefficient which matches the findings of [14]. Fig.4 compares the streamtraces, vorticity field and tracer field in the near wake region of the cylinder. The tracer field in generated by virtue of the group injection described previously. The alternate detachment of vortices from the cylinder wall and subsequent downstream advection, results in the generation of a wave-like pattern in the Eulerian description of the flow. The amplitude of these \textit{Eulerian waves} gets damped as one move downstream which is observable from fig. 4(a). Subsequently, the streamlines become parallel to the x-axis. Fig. 4(b) shows the contours of vorticity magnitude indicating the alternate detachment of vortices in the downstream direction. The rotationality of the regime attenuates rapidly by virtue of viscous dissipation as the vortices are advected downstream. This explains the progressive damping of the waves in the Eulerian realization. The influence of the shed vortices is felt up to a maximum downstream distance of 90D.

Fig. 4. The shedding regime at Re=100, (a) A wavy streamline pattern formed in the Eulerian realization, (b) Line contours of vorticity magnitude (in s$^{-1}$) showing downstream advection and subsequent viscous dissipation of vortices, (c) Particle tracks (colored by injection ID) formed by the group injection of neutrally buoyant tracers showing the onset of Lagrangian chaos and (d) Approximate 2D soap film visualization of [15] for a Reynolds number falling within Roshko’s stable shedding range.

Fig. 4(c) shows the particle tracks made by the neutrally buoyant tracers. Once introduced, the particles are quickly influenced by the \textit{sweeping action} of the vorticity field. Globally, the particle tracks show an excellent match with the vorticity field. However, after traversing a downstream distance of 10D, the tracer field experiences an onset of irregularity. This is marked by an increasing rate of convolution in the particle trajectories. A closer look at the tracks reveals that the interconnecting 'arms' (fig. 4(c)) also undergo considerable amount of distortion and elongation as the regime progresses downstream. A single particle kept on such a trajectory will be required to travel an 'ever increasing' distance to move from one vortical region to another. Also, the extent of mixing between particle streams is remarkably good following the onset of chaos. The trajectories hence depict a situation in which the tracers move through a very slowly dissipating vorticity field which is not in accordance with the streamline pattern of fig. 4(a). Looking at the particle trajectories, one may get an impression that the strength of the shed vortices persists well beyond that predicted by the viscous dissipation of vorticity, the tracers indicating a sort of
structure in the velocity field even when there is none. The results hence indicate a case similar to what was observed by [11]. Fig. 4(d) shows a similar visualization by [15] using a soap-film tunnel.

Fig. 5 depicts the fate of the particle trajectories beyond a downstream distance of 50D. It can be observed that the convolution rate of trajectories as well as tracer mixing have increased significantly. The zoomed-in detail shows the extent of particle meandering in a pair of vortical regions having opposite rotationality. The detail also depicts ‘islands’ of fluid in an otherwise convoluted scatter of tracers. This is similar to what Ottino [17] observed in the time dependent flow between eccentric cylinders. Thus, the tracer behavior has some global similarities which transcend a range of time-dependent flows. The islands are observed to ‘grow’ as the regime progresses downstream.

![Fig. 5. Particle tracks (colored by injection ID) in a region 50D downstream of the cylinder, (a) Large-scale convoluted structures (numerical blobs) increasing in complexity and extent of mixing and (b) Zoomed-in view of a pair of numerical blobs showing mixing of different injection streams.](image)

As correctly mentioned by [18], given the downstream tracer distribution of fig. 5; it is impossible to find the sources of agitation. The fact that the accuracy of any tracer visualization strongly depends on the location of injection is firmly established by fig. 6. Here, the emission of particles from injectors 1 and 2 is considered. In case of injector 1, the trajectory becomes chaotic nonetheless, but the onset of chaos is delayed. The initial agreement with the streamlines is good; but the trajectory rapidly deviates from linearity as the tracers move downstream. Subsequent tracks are largely characterized by the nucleation of highly curved, cliff-like patterns. This is shown in figure 6(a). An emission made from injector 2 (located 30D downstream of injector 1) indicates an even greater delay in the onset of chaos. The amplitude of particle meandering shows a gradual rise as the tracers get advected downstream by the mean flow (fig. 6(a)). The tracer field is hence expected to become completely organized with the streamlines if the injector position is shifted further downstream. Thus, the results obtained until now indicate that for Re=20, the streamtraces and streaklines coincide for a group injection upstream of the cylinder. But this is not the case for Re=100, where multiple injections at different locations in the regime depict varying particle behavior. The following sub-

![Fig. 6. Particle tracks (colored by particle residence time) formed by point injections located downstream of the cylinder, (a) Injector 1 (at 50D) shows a slight delay in the onset of chaos; the extent of delay being higher in the case of injector 2 (at 80D) and (b) Corresponding streamlines in the region.](image)

sections deal with the mathematical treatment of the cases discussed so far.

3) Chaotic Advection The particles are termed as ‘passive’ because, once released, they have no choice but to follow the fluid flow. Having no momentum of their own, when released, these particles act as streaklines and exhibit what can be termed as passive advection. In this case the Eulerian and Lagrangian realizations of the regime are exactly the same (case 1). Further, any passive tracer placed in a fluid flow has to fulfill one condition: at all points within the configuration space; the velocity of the particle will be equal to the flow. Mathematically, the above phenomenon results into the advection equations [17]-[20], which are obtained for a three dimensional Euclidean space as follows,

\[ V_{\text{particle}} = V_{\text{fluid}} \]

\[ V_{\text{particle}} = \left( \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \right) \]

Equation (4) emphasizes the fact that the velocity of a particle is the rate of change of its position considering the Lagrangian realization. Similarly, in the Eulerian sense, the fluid velocity is defined as,

\[ V_{\text{fluid}} = \{ u(x,y,z,t), v(x,y,z,t), w(x,y,z,t) \} \]

Comparing equations (4) and (5), a set of differential equations can be formulated as,

\[ \frac{dx}{dt} = u(x,y,z,t), \frac{dy}{dt} = v(x,y,z,t), \frac{dz}{dt} = w(x,y,z,t) \]

(6)

The equations comprising set (6) are termed as the advection equations which govern the behavior of a passive scalar placed within a fluid flow. For the present case, the
space is two dimensional and hence the equation set (6) simplifies to,
\[
\frac{dx}{dt} = u(x,y,t), \quad \frac{dy}{dt} = v(x,y,t)
\]  
(7)
A two dimensional flow regime is associated with the stream-function, $\psi$, which is defined as,
\[
u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}
\]  
(8)
Finally, the equation sets (5) and (6) result into,
\[
\frac{dx}{dt} = \frac{\partial \psi}{\partial y} \quad \frac{dy}{dt} = -\frac{\partial \psi}{\partial x}
\]  
(9)
The equation set (9) is equivalent to Hamilton’s canonical equations (for a unit degree of freedom system) with $\psi$ being the Hamiltonian [19]. Hamiltonian dynamics and dynamic systems theory state that Hamilton’s equations are bound to produce chaotic solutions. This is the case when a system is dependent on three variables; two spatial and one temporal. Applying this theory to the results obtained for case 1, it is natural to state that the advection equations do not produce chaotic solutions for a two dimensional flow regime that is temporally invariant. The case of flow over a circular cylinder at Re=20 symbolizes a velocity field that is only dependent on spatial dimensions (x and y dependent). Since there is no dependence on a “third” temporal variable, the solutions are not chaotic, and hence, the streakline pattern coinciding with the streamlines is justified. At Re=100, however, the shedding regime establishes the dependence of the velocity field on three variables. The flow, being periodic, now varies with two spatial variables and time. The advection equations are hence rendered non-integrable in the phase space of the corresponding Hamiltonian system. The chaotic solutions produced within the phase space naturally propagate into the configuration space of the advected particle. The flow field at Re=20 is time invariant and comprises of a pair of standing vortices behind the cylinder. The particles, hence, do not exhibit chaotic behavior in this case and form regular spatial trajectories. The case involving vortex shedding at Re=100 exhibits a completely different tracer behavior. The advection equations, now being time dependent, produce chaotic solutions which are reflected in the particles’ tendency to form random spatial distributions. The extent of clusters produced within the particle tracks is also heavily dependent on the amount of memory of the upstream flow that the particles carry along. Hence, the location of the particle injection becomes crucial if a correct visualization of the velocity field is to be obtained. The case at Re=100 hence becomes a classical example of a flow regime in which the streamlines and streaklines do not coincide. Hence, injecting a passive tracer just downstream of a body placed in a transient flow may lead to a realization that is misleading. It is thus advisable to inject particles at multiple locations in order to obtain a correct visualization of the regime.

IV. SUMMARY

Numerical flow visualization is carried out by introducing passive tracers into the steady and periodic wakes of a circular cylinder placed in cross-flow. Two cases are studied: one at a Reynolds number of 20 and the other at Re=100. The behavior of a passive tracer in a flow regime is governed by the advection equations based on the fact that the velocity of the particle (Lagrangian realization) is equal to that of the flow (Eulerian realization). If the Eulerian velocity field is dependent on more than two variables (out of three spatial and one temporal), the set of advection equations becomes equivalent to a single degree of freedom Hamiltonian system and produces chaotic solutions in the configuration space of the advected particle. The flow field at Re=20 is time invariant and comprises of a pair of standing vortices behind the cylinder. The particles, hence, do not exhibit chaotic behavior in this case and form regular spatial trajectories. The case involving vortex shedding at Re=100 exhibits a completely different tracer behavior. The advection equations, now being time dependent, produce chaotic solutions which are reflected in the particles’ tendency to form random spatial distributions. The extent of clusters produced within the particle tracks is also heavily dependent on the amount of memory of the upstream flow that the particles carry along. Hence, the location of the particle injection becomes crucial if a correct visualization of the velocity field is to be obtained. The case at Re=100 hence becomes a classical example of a flow regime in which the streamlines and streaklines do not coincide. Hence, injecting a passive tracer just downstream of a body placed in a transient flow may lead to a realization that is misleading. It is thus advisable to inject particles at multiple locations in order to obtain a correct visualization of the regime.

References

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